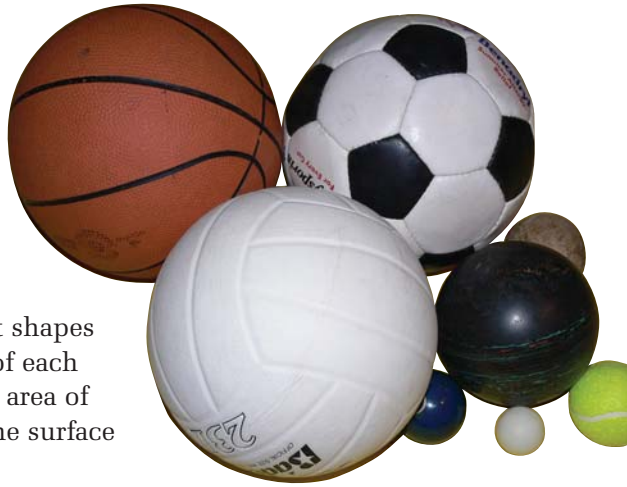
8.6 Surface Area of a Sphere

sphere

- a round ball-shaped object
- a set of points in space that are a given distance (radius) from a fixed point (centre)

A **sphere** is a three-dimensional shape that is often seen in sports. Balls of different sizes are used to play basketball, soccer, and volleyball, to name a few.



Consider the balls shown. What shapes appear to make up the surface of each sphere? How could you use the area of these shapes to help you find the surface area of a sphere?

Investigate

Tools

- orange
- string
- ruler
- centimetre grid paper

How can you model the surface area of a sphere?

Work with a partner.

1. Choose an orange that is as spherical as possible. Estimate the surface area of the orange, in square centimetres.

2. **a)** Measure the circumference of the orange. Use a piece of string to go around the outside of the orange. Then, measure the length of the string.

   b) Use the formula for the circumference of a circle, $C = 2\pi r$, to find the radius of the orange.

3. Carefully peel the orange. Flatten the pieces and place them on grid paper. Trace around the pieces. Find the area of the peel by counting squares and partial squares on the grid paper.

4. **a)** Determine the area of a circle with the same radius as the orange.

   b) What is the approximate ratio of the orange's surface area to the area of the circle?

   c) Describe a possible formula for the surface area of a sphere based on your results.

5. **Reflect** Compare your results with those of your classmates. What do you conclude that the formula for the surface area of a sphere is?

Did You Know?

The first in-socket artificial eyes were made in the 15th century from gold and coloured enamel. Today they are made from a medical grade acrylic plastic.

Example 1 Surface Area of an Eyeball

The dimensions of an adult human eyeball are reasonably constant, varying only by a millimetre or two. The average diameter is about 2.5 cm. Calculate the surface area of the human eyeball, to the nearest tenth of a square centimetre.

Solution

The formula for the surface area of a sphere is $SA_{\text{sphere}} = 4\pi r^2$.

The eyeball has a diameter of 2.5 cm, so the radius is 1.25 cm.

$$\begin{aligned} SA_{\text{sphere}} &= 4\pi r^2 \\ &= 4\pi(1.25)^2 \quad \text{Estimate: } 4(3)(1)^2 = 12 \\ &\doteq 19.6 \end{aligned}$$

The surface area of the human eyeball is about 19.6 cm^2.

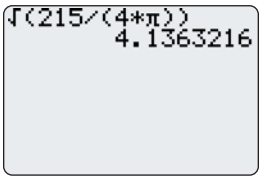
Example 2 Find the Radius of a Baseball

Determine the radius of a baseball that has a surface area of 215 cm^2. Round your answer to the nearest tenth of a centimetre.

Solution

Substitute the values into the formula.

$$\begin{aligned} SA_{\text{sphere}} &= 4\pi r^2 \\ 215 &= 4\pi r^2 \\ \frac{215}{4\pi} &= r^2 \\ \sqrt{\frac{215}{4\pi}} &= r \\ r &\doteq 4.1 \end{aligned}$$

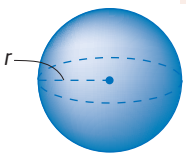


C 215 ÷ (4 × π) = √

The radius of the baseball is about 4.1 cm.

Key Concepts

- The formula for the surface area of a sphere with radius r is $SA_{\text{sphere}} = 4\pi r^2$.
- If you know the surface area of a sphere, you can determine the radius, r, of the sphere.



Communicate Your Understanding

C1 Describe how you would determine the amount of leather required to cover a softball.

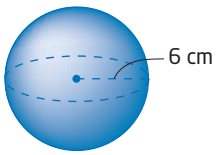
C2 Does doubling the radius of a sphere double the surface area? Explain your reasoning.

Practise

For help with questions 1 and 2, see Example 1.

1. Determine the surface area of each sphere. Round to the nearest square unit.

a)



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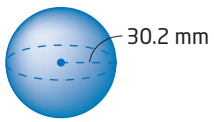

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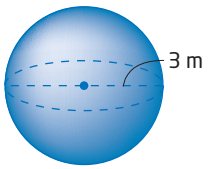

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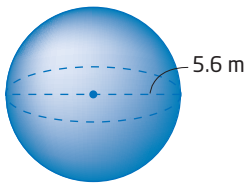
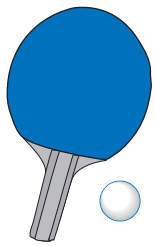


2. A ball used to play table tennis has a diameter of 40 mm.

a) Estimate the surface area of this ball.

b) Calculate the surface area, to the nearest square millimetre. How close was your estimate?



For help with question 3, see Example 2.

3. A sphere has a surface area of 42.5 m^2. Find its radius, to the nearest tenth of a metre.

Connect and Apply

4. A basketball has a diameter of 24.8 cm.

a) How much leather is required to cover this ball, to the nearest tenth of a square centimetre?

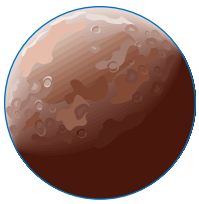
b) If the leather costs \$28/m^2, what does it cost to cover the basketball?

5. The diameter of Earth is approximately 12 800 km.

a) Calculate the surface area of Earth, to the nearest square kilometre.

b) What assumptions did you make?

6. **a)** The diameter of Mars is 6800 km. Calculate its surface area, to the nearest square kilometre.



b) Compare the surface area of Mars to the surface area of Earth from question 5. Approximately how many times greater is the surface area of Earth than the surface area of Mars?

7. **Chapter Problem** Emily is placing a gazing ball in one of her customer's gardens. The ball has a diameter of 60 cm and will be covered with reflective crystals. One jar of these crystals covers 1 m^2.

a) Estimate the surface area to decide whether one jar of the crystals will cover the ball.

b) Calculate the surface area, to the nearest square centimetre.

c) Was your estimate reasonable? Explain.

8. The radius of a sphere is 15 cm.

a) Predict how much the surface area increases if the radius increases by 2 cm.

b) Calculate the change in the surface area, to the nearest square centimetre.

c) How accurate was your prediction?

9. **Use Technology**

a) Use a graphing calculator to graph the surface area of a sphere versus its radius by entering the surface area formula.

b) Describe the relationship.

c) Use the TRACE feature to determine
- the surface area of a sphere with radius 5.35 cm
- the radius of a sphere with surface area 80 cm^2

Extend

10. **Use Technology**

a) Determine an algebraic expression for the radius of a sphere in terms of its surface area.

b) Use your expression from part a) and a graphing calculator to graph the relationship between the radius and the surface area.

c) Describe the relationship.

d) Use the graphing calculator to find the radius of a sphere with surface area 200 cm^2.

11. A spherical balloon is blown up from a diameter of 10 cm to a diameter of 30 cm. By what factor has its surface area increased? Explain your reasoning.

12. Which has the larger surface area: a sphere of radius r or a cube with edges of length $2r$?

13. Use Technology A sphere just fits inside a cube with sides of length 10 cm.

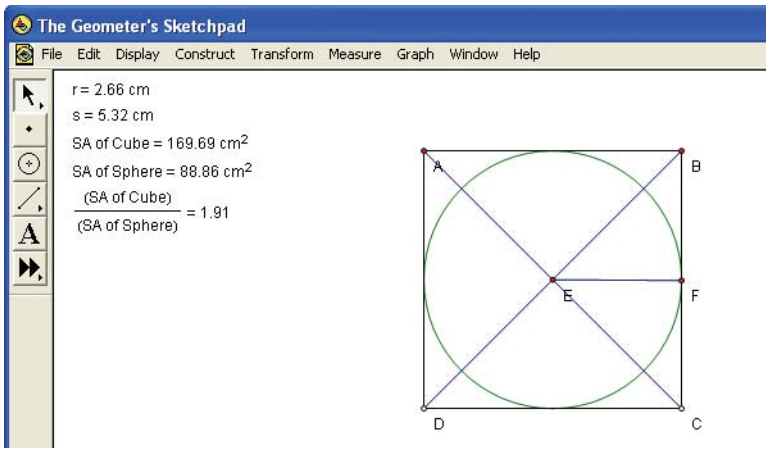
a) Estimate the ratio of the surface area of the sphere to the surface area of the cube.

b) Calculate the surface areas of the sphere and the cube and their ratio.

c) How does your answer compare to your estimate?

d) Use *The Geometer's Sketchpad*® to investigate this relationship for any size of cube with an inscribed sphere. Since *The Geometer's Sketchpad*® cannot easily show three-dimensional objects, represent the cube with a square and the sphere with a circle.

- From the **Edit** menu, choose **Preferences**. Click on the **Text** tab. Ensure that **For All New Points** is checked.
- Select the **Custom Tool**. From the drop-down menu, choose **Polygons** and then **4/Square (By Edge)**. Draw a square ABCD.
- Construct the diagonals of the square. Draw a circle with its centre at E, where the diagonals cross, such that it is inscribed in the square. Draw a radius EF.
- Measure radius EF of the circle. Select this measurement. Right click and choose **Label Measurement** from the drop-down menu. Change the label to **r**.
- Measure side AB of the square. Change the label to **s**.
- Select **s**. From the **Measure** menu, choose **Calculate**. Enter the formula 6*__s__^2 by selecting **s** from the **Values** drop-down menu on the calculator. Change the label to **SA of Cube**.
- Select **r**. From the **Measure** menu, choose **Calculate**. Enter the formula 4*π*__r__^2 by selecting **r** from the **Values** drop-down menu on the calculator. Change the label to **SA of sphere**.
- Calculate the ratio $\frac{\text{SA of cube}}{\text{SA of sphere}}$.



- Drag point A. Watch how the measurements change.

What can you conclude about the ratio of the surface areas of a cube and a sphere inscribed in the cube?